

An ASABE Meeting Presentation

Paper Number: 072117

Modeling Streambank Instability by Seepage Undercutting

Maria Librada Chu-Agor, Ph.D. Student

Department of Biosystems and Agricultural Engineering, Oklahoma State University, 209 Agricultural Hall, Stillwater, OK 74078-6016, ma.librada.agor@okstate.edu

Glenn Wilson, Ph.D., Hydrologist/Soil Scientist

USDA-ARS National Sedimentation Laboratory, 520 McElroy Drive, Oxford, MS, 38655, gvwilson@msa-ars.usda.gov

Garey A. Fox, Ph.D., Assistant Professor

Department of Biosystems and Agricultural Engineering, Oklahoma State University, 120 Agricultural Hall, Stillwater, OK 74078-6016, garey.fox@okstate.edu

Written for presentation at the 2007 ASABE Annual International Meeting Sponsored by ASABE Minneapolis Convention Center Minneapolis, Minnesota 17- 20 June 2007

Abstract. Predicting bank collapse due to seepage erosion undercutting has not been fully studied or modeled, even though its role in streambank erosion may be important. The limitation originates from the lack of field measurements or laboratory experiments as well as the unavailability of discrete element models that can effectively simulate seepage erosion. The objective of this research was to demonstrate a procedure for incorporating seepage undercutting into two-dimensional variably-saturated flow and bank stability models and to investigate the role of seepage undercutting on bank instability. Integrated flow and bank stability models were used to simulate soil-water pressure variations and bank stability with and without seepage erosion with regard to input parameter uncertainty using Monte Carlo analysis. The percentage decrease in the mean factor of safety, Fs, ranged between 42 and 91% as the depth of undercutting increased, dependent upon the initial stability of the bank. For stable banks, the probability of failure reached 100% when the depth of the undercutting reached 30 to 50 mm. The propensity of streambanks to fail during the recession limb of hydrographs may be the combined result of seepage undercutting and reduced cohesion.

Keywords. Bank Stability, Sapping, Seepage Erosion, Streambank Failure, Undercutting

The authors are solely responsible for the content of this technical presentation. The technical presentation does not necessarily reflect the official position of the American Society of Agricultural and Biological Engineers (ASABE), and its printing and distribution does not constitute an endorsement of views which may be expressed. Technical presentations are not subject to the formal peer review process by ASABE editorial committees; therefore, they are not to be presented as refereed publications. Citation of this work should state that it is from an ASABE meeting paper. EXAMPLE: Author's Last Name, Initials. 2007. Title of Presentation. ASABE Paper No. 07xxxx. St. Joseph, Mich.: ASABE. For information about securing permission to reprint or reproduce a technical presentation, please contact ASABE at rutter@asabe.org or 269-429-0300 (2950 Niles Road, St. Joseph, MI 49085-9659 USA).

Introduction

The erosion of sediment from streambanks can be overwhelmingly dominated by mass wasting (Simon and Darby, 1997). Subsurface erosion is often regarded as a process of limited importance confined to certain soils and streambank stratigraphies (Bryan and Jones, 1997). The effect of seepage or subsurface flow is usually considered to be limited to the production of surface runoff, thereby underestimating the potential effects of seepage on erosion (Owoputi and Stolte, 2001). Due to the lack of knowledge and the general opinion that seepage effects are small, especially compared to other processes and forces involved, seepage effects are generally neglected in stream channel designs (Burgi and Karaki, 1971). The significance of seepage erosion, called sapping, has not been widely recognized or understood despite the documentation of its occurrence in numerous geographical locations (Hagerty, 1991).

It is widely recognized that seepage reduces bank stability and thus cohesive strength by increasing soil-water pressure (Abam, 1993; Darby and Thorne, 1996; Crosta and Prisco, 1999; Rinaldi and Casagli, 1999; Simon et al., 1999). Burgi and Karaki (1971) developed an empirical relationship between the seepage forces acting on the side slope of a channel and the stability of the channel with various flow conditions. They confirmed that side slopes with seepage were less stable than channels without seepage. Crosta and di Prisco (1999) studied seepage erosion causing liquefaction and rapid slope failures by comparing observed field failure mechanisms and the evolution of the saturated domain using a numerical model. They reported that failure was induced by the three dimensional development of the saturated domain from a localized source. Hagerty et al. (1981) investigated bank erosion in the Ohio River and concluded that one of the principal erosion mechanisms is internal erosion of bank materials by discharge following floods. Kusakabe et al. (1987) carried out a series of centrifuge model tests to study river bank failures due to seepage flow and found that clay and silt content of river bank material have significant effect on the importance of seepage flow. Dapporto et al. (2003) investigated the mechanisms of failure and retreat of Arno River in Italy and demonstrated that the complex interaction between the loss of soil strength by increased soil-water pressures and the temporal stability of the confining pressures during high river stage plays a primary role in triggering mass failures.

According to Rockwell (2002), the greatest weakness of both seepage and soil-water pressure studies has been the lack of direct, local and precise instrumentation. Quantitative data are not available at the point of erosion, and quantitative existence of seepage is only known indirectly. This limitation could be due to the difficulty of conducting field studies during wet periods when seepage is active (Huang and Laflen, 1996; Wilson et al., 2007). Study of failure due to seepage flow requires accumulated data of close observations on the phenomena in the field as well as laboratory reproduction of these phenomena.

A few studies in the literature have begun to study seepage erosion in the laboratory and field with the detail suggested by Rockwell (2002). Lourenco et al. (2006) examined the relation between soil-water pressure and the failure mode at the interface of two soil layers of different permeability. Although their experiments did not show any clear relation between soil-water pressure and the failure mode, it demonstrated that seepage strongly controlled the failure mechanisms. Fox et al. (2006) and Wilson et al. (2007) conducted lysimeter experiments of the undercutting of streambanks by seepage flow indicating that seepage undercutting, independent of the loss of negative soil-water pressure, could result in bank collapse. Wilson et al. (2007) documented the first in-situ detailed measurements of seepage flow, erosion, and bank undercutting and demonstrated that streambank stratigraphy and layering were important in controlling seepage flow. Fox et al. (2007) demonstrated that undercutting occurs not only due

to seepage through a conductive, noncohesive streambank layer but can also occur when seepage erosion undercuts less cohesive layers underneath the conductive layer.

While studies quantifying the effects of seepage erosion on bank stability are improving, a bank stability analysis has not yet been developed to address the effects of bank undercutting brought about by seepage erosion. Few, if any, studies on seepage have attempted to incorporate bank instability by the combined forces of increased soil-water pressure and seepage undercutting. Wilson et al. (2007) acknowledged the need to incorporate a subsurface flow model with a streambank stability model and suggested that the dynamic process of seepage erosion and undercutting needs to be included in the combined models. The objective of this research was to develop and demonstrate a procedure that will quantify the effects of seepage undercutting on bank stability. The two-dimensional lysimeter experiments of Fox et al. (2006) and Wilson et al. (2007) were simulated using variably-saturated flow and bank stability models.

Materials and Methods

Lysimeter Experiments

Fourteen lysimeter experiments were performed by Fox et al. (2006) and Wilson et al. (2007) to simulate seepage erosion at Little Topashaw Creek (LTC) in northern Mississippi. The simulated LTC streambanks consisted of a silt loam (SiL) top soil of varying bank height, a 0.10 m conductive loamy sand (LS) layer, and a 0.05 m clay loam (CL) restrictive layer at the bottom (Figure 1). Flow through the lysimeter was controlled using constant heads of 0.3, 0.4, 0.6, and 0.9 m. The base of the lysimeter was tilted to simulate sloping banks of 0%, 5%, and 10% slopes. Of the fourteen lysimeter experiments performed, several were excluded in the modeling due to development of horizontal shear failure planes as a result of the packing. The six lysimeter experiments studied in this research included bank heights (BH) that varied from 0.3 to 0.8 cm, constant heads (H) that varied from 0.3 to 0.9 m, and slopes (S) from 0 to 10% (Table 1). The simulations also included an experiment where bank failure did not occur despite seepage undercutting (H = 0.3 m, BH = 0.4 m, and S = 0%).

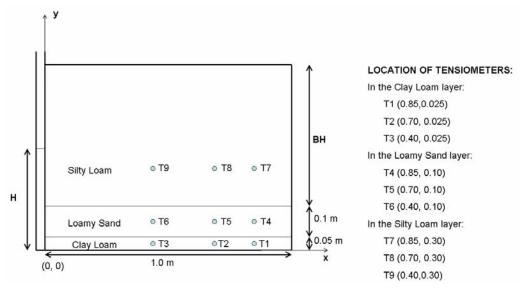


Figure 1. Lysimeter set-up showing the location of the tensiometers and the hydraulic controls of the experiment.

Data from the lysimeter experiments included soil-water pressure measured by nine tensiometers (Figure 1) within the three streambank layers and cumulative discharge measurements at specific times (Periketi, 2005; Wilson et al., 2007). During the lysimeter experiments, the depth of seepage undercutting, referred to as the horizontal distance from the drainage face of the lysimeter into the bank, was measured. These measurements were used to simulate seepage undercutting in the bank stability model.

Table 1. Boundary conditions for the two-dimensional seepage erosion lysimeter experiments simulated using SEEP/W and SLOPE/W.

Constant Head, H	Bank height, BH	Slope, S
(m)	(m)	(%)
0.3	0.4	0
0.3	0.8	0
0.6	0.5	0
0.6	0.8	5
0.6	0.8	10
0.6	0.5	10

Variably-Saturated Flow Modeling

The lysimeter experiment was modeled using SEEP/W to simulate the variations in the soil-water pressure and cumulative discharge. SEEP/W is a finite element model of Richards' equation for two-dimensional variably-saturated flow (Krahn, 2004a). The flow domain was constructed to represent the geometry of the lysimeter with five or six internal material regions. The CL layer was considered as one region, the SiL another region and the conductive LS layer was divided into three or four regions to facilitate the change in the flow domain geometry to account for seepage erosion undercutting the streambank. The regions were then discretized into 25 by 25 mm elements.

Specifying and assigning material properties in SEEP/W involves defining the water retention function, $\theta(h)$, and the hydraulic conductivity function, K(h), where h is the soil-water pressure (Krahn, 2004a). It is a common practice to use an estimation method to represent $\theta(h)$, such as the van Genuchten (1980) model. Field measurements of soil hydraulic properties were used to define the parameters of the van Genuchten (1980) model. Soil samples were taken from LTC field sites where seepage was occurring through the conductive LS layer (Fox et al., 2006; Wilson et al., 2007). These values were used as default soil hydraulic property values prior to calibration. SEEP/W can represent K(h) by the van Genuchten (1980) model using parameters obtained from the water retention function and a saturated hydraulic conductivity value, K_s . Calibration of the models was carried out by slightly adjusting K_s as well as the van Genuchten curve-fitting parameters (α , m, and n).

SEEP/W uses either Dirichlet or Neuman boundary conditions in which the hydraulic head or the discharge, respectively, is specified at a boundary. If the discharge is specified, SEEP/W will compute the soil-water pressure to maintain the specified discharge and vice versa. The initial conditions of the models were derived from the initial measured soil-water pressure from the lysimeter experiments. A potential seepage review boundary condition for all the nodes was assigned at the drainage face. In SEEP/W, a potential seepage review boundary condition is used when neither the hydraulic head nor the discharge are known beforehand but instead must

be computed by the model (Krahn, 2004a), as in the case of the drainage from the lysimeter or bank face. A hydraulic boundary function was used as the boundary condition at the inflow face and a zero flux boundary condition was specified for the top and bottom boundaries of the flow domain.

The performance of the SEEP/W models was quantified by using an objective function and by visual comparison of the simulated and measured soil-water pressure and cumulative discharge. Differences between the *n* simulated and observed cumulative discharge values were minimized based on linear regression while also minimizing the root mean square error (RMSE) of the simulated and measured soil-water pressure.

Streambank Stability Modeling

SLOPE/W is a numerical slope stability model which uses the theory of limit equilibrium of forces and moments to compute the factor of safety, *Fs*, against failure. It involves discretizing a potential sliding mass into vertical slices and applying equations of statics (Krahn, 2004b). *Fs* is defined as that factor by which the shear strength of the soil must be reduced in order to bring the mass of soil into a state of limiting equilibrium along a selected slip surface. The *Fs* is an index of the relative stability of a slope, with values less than 1.0 indicating failure.

SLOPE/W was used to analyze the stability of the streambank as simulated by the lysimeter experiments. The stability modeling procedure had three components: (1) definition of the geometry and shape of the potential slip surface, (2) definition of the soil strength properties, and (3) definition of the soil-water pressure. SEEP/W and SLOPE/W are integrated codes such that the geometry defined in SEEP/W is used in SLOPE/W. Soil strength parameters in the lysimeter experiment were defined using Coulomb's equation. For an effective stress analysis, the shear strength is defined as:

$$s = c' + (\sigma_n - u) \tan \phi' \tag{1}$$

where s = shear strength, c' = effective cohesion, ϕ' = effective angle of internal friction, σ_n = total normal stress, and u = soil-water pressure (Krahn, 2004b). The Morgenstern-Price (1965) method was selected for computing Fs. This method satisfies both the moment and force equilibrium equations and can give accurate results for all practical conditions (Duncan and Wright, 1980; Krahn, 2004b).

The soil-water pressures generated from SEEP/W were input into SLOPE/W. The model was then run using the soil-water pressure distribution at every time step to determine the effect of the changes on the stability of the slip surface. The auto-search option was chosen for defining the potential slip surface. In this method, SLOPE/W generated 1000 trial slip surfaces to find the most probable minimum slip surface based on the problem's geometry by identifying the most probable entry and exit areas of the slip surface. This method can result in unrealistic slip outputs so that comparison of the generated slip surface with the actual appearance of the collapsed bank is necessary.

For each lysimeter experiment, a probabilistic slope stability approach of solving the *Fs* was adopted by considering the variability of the soil strength parameters of the SiL and LS layers. SLOPE/W can perform a probabilistic slope stability analysis which allows for the consideration of variability in input parameters (Krahn, 2004b). The user can assign a probability density function (*pdf*) to input parameters (Caviness et al., 2006). Using the specified *pdf*, SLOPE/W derives the cumulative distribution function by integrating the area under the *pdf*. The cumulative distribution function is then inverted to produce the sampling function. Each time a random number is generated from Monte Carlo method, the parameter is "sampled" using this

function. The randomly generated parameter is then fed into the deterministic model to compute the *F*s.

Field measurements of cohesion, angle of internal friction, and total unit weight from the LTC streambank site where the lysimeter soil was sampled were carried out using a borehole shear test at two field locations. Average soil strength values, Table 2, were used to define the material properties of the layers for the slope stability model. The variability in these soil strength parameters was assumed to follow a normal probability density function similar to most geotechnical engineering material properties (Krahn, 2004b). A standard deviation equal to 2.0 was chosen and 2000 Monte Carlo trials were simulated as suggested by Krahn (2004b).

Table 2. Soil strength parameters of Little Topashaw Creek (LTC) streambank based on measurements at two sites where seepage erosion was observed. Values for cohesion and angle of internal friction are average values used in the SLOPE/W model. Parameter values in parentheses are values from each site.

Layer	Depth (m)	Cohesion (kPa)	Angle of Internal Friction (°)	Total Unit weight (kN/m³)
Silt loam (SiL)	0.5	7.5 (5.0, 10.0)	30.0 (25.0, 35.0)	16.0
Loamy sand (LS)	1.5	1.0 (1.0, 1.0)	25.5 (22.0, 29.0)	19.0
Clay Loam (CL)	2	15.0 (15.0)	35.0 (35.0)	21.0

The unavailability of models to simulate undercutting, which modifies the flow domain with time, makes it difficult to quantify the importance of seepage erosion in slope stability analysis. SEEP/W uses a finite element method which requires the elements to be connected at the corners by nodes which is not representative of an undercutting process where the elements tend to "break-away" from the adjacent elements. This is a limitation of a finite element method like SEEP/W. SLOPE/W on the other hand, being a limit equilibrium program, cannot model over hanging walls or undercut slopes where the base of some slices are exposed to the air and shear forces cannot be computed for these slices. This is the case of undercutting brought about by seepage erosion. In order to overcome these limitations, a procedure was developed to incorporate the effects of undercutting into the variably-saturated flow stability models.

For the lysimeter experiments, seepage erosion was simulated by manually changing the geometry of the LS layer based on available data for the dimensions and shape of the undercutting. The shape, dimensions and timing of undercutting were measured during the lysimeter experiment as reported by Periketi (2005) and Wilson et al. (2007). From this information, seepage erosion was modeled by dividing the LS layer into segments. Changes in the geometry of the domain to reflect the shape and location of the undercutting was accomplished by changing the material properties of segments. SLOPE/W's limitation regarding undercutting was addressed by covering the cut with a null region without specifying any soil strength properties. In SEEP/W, this region was treated as a void in the flow domain by not assigning a material property. This will exclude the weight of the null region in the analysis. The performance of the SLOPE/W models in predicting the shape of the critical slip surface was

evaluated by comparing the measured dimensions of the collapsed bank against the critical slip surface generated by the model.

Results and Discussion

Soil-water pressure generated from SEEP/W at specific time steps were used to define the input soil-water pressure in SLOPE/W. The predicted, mean *Fs* when undercutting was not considered, Figure 2, did not significantly change during any of the lysimeter simulations. Changes in soil-water pressure did not sufficiently affect soil strength in these experiments and therefore did not reduce the stability of the bank. Yet it is the impact of soil-water pressure on soil strength that is most often attributed to bank failure (e.g., Burgi and Karaki, 1971; Edil and Vallejo, 1980; Darby and Thorne, 1996; Abam, 1993; Crosta and Prisco, 1999; Rinaldi and Casagli, 1999; Simon et al., 1999; Dapporto et al., 2003). In contrast to the impact of soil-water pressure, the *Fs* decreased approximately 42% (from a mean value of 1.06 to 0.62) and 55% (from a mean value of 1.05 to 0.47) for experiments with 0.8 m BH, 0% S, and 0.3 and 0.6 m H, respectively, when seepage undercutting was considered (Figure 2). This resulted in an unstable bank (i.e., *Fs*<1.0) at the end of the simulation. Experiments with greater than 0% slope were predicted to be unstable at the beginning of the simulation based on the mean *Fs* (*Fs*<1.0). The *Fs* for these experiments decreased 50 to 91% between simulations when seepage undercutting was included.

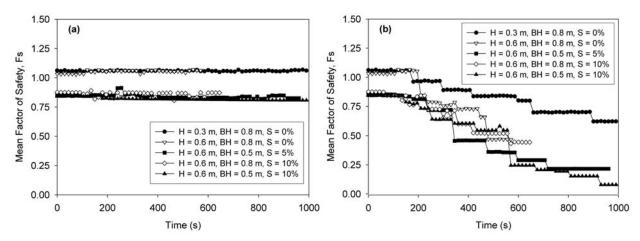


Figure 2. Mean factor of safety (Fs) versus time as predicted by SLOPE/W Monte Carlo analysis for lysimeter experiments (a) without considering seepage undercutting and (b) with seepage undercutting.

For stable banks with sufficient undercutting measurements (i.e., 0.8 m BH, 0% S), the change in the probability of failure, or level of risk of instability, increased by approximately 180% (from 35.2% to 100%) for H=0.3 m and 170% (from 36.5% to 100%) for H = 0.6 m when seepage undercutting became active (Figure 3). The probability of failure is determined by counting the number of *Fs* values less than 1.0 with respect to the total number of converged slip surfaces. It is equivalent to the percentage of slopes that would fail if a slope were to be constructed repeatedly. A probability of failure equal to 100% was reached when the depth of undercutting reached approximately 30 to 50 mm into the bank. Since the parameters used in the simulation with and without seepage erosion were the same, the increase in the probability of failure and

decrease in the *Fs* can be attributed to the change in the geometry of the LS layer due to simulated undercutting by seepage erosion. The results also show that a stable bank (i.e., *Fs*>1.0) can become significantly unstable when seepage erosion is considered.

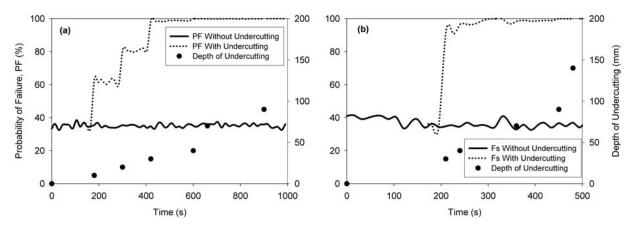


Figure 3. Simulated probability of failure (PF, %) of lysimeter experiments with 0.8 m bank, 0% slope, and (a) 0.3 m constant head and (b) 0.6 m constant head with and without seepage undercutting.

The SLOPE/W model consistently predicted failure ahead of the observed bank collapse (i.e., a 100% probability of failure was reached before the actual collapse observed during the lysimeter experiments). For example, SLOPE/W predicted collapse approximately 300 to 500 s before actual bank collapse for the lysimeter experiments with 0.8 m BH, 0% S, and 0.3 and 0.6 H. For the experiment where bank failure did not occur due to seepage undercutting (H = 0.3 m, BH = 0.4 m, and S = 0%), the model predicted a bank collapse of 0.05 m 3 . This is an indication that the lysimeter setup is more stable than the bank simulated by the model. This condition can be due to two factors. First, the walls of the lysimeter (only separated by 15 cm) induced compressive forces to counteract stresses produced by the soil weight. Also, uniform packing of disturbed soil samples can add extra strength to the bank relative to natural field heterogeneity. Even with these experimental conditions, the model predictions of bank collapses were generally within 33% of the measured volume of bank collapse.

The depth of undercutting and the corresponding mean *Fs* were evaluated by grouping the six experiments into four categories. The minimum and maximum values of the *Fs* were also determined. The first category consists of experiments with the same BH and S but different H. As theoretically expected, these experiments possess the same initial stability. However, as time increased, the bank with higher H had lower *Fs* at a given time due to the fact that the higher H resulted in more rapid undercutting of the bank by seepage erosion (Figure 2). However, the same depth of undercutting resulted in approximately the same bank stability or mean *Fs* (Figure 4a). The second category consists of experiments with the same H and S but different BH, which possess different initial bank stabilities: the bank with BH=0.4 m was initially 39% more stable than BH=0.8 m. However, as undercutting progressed, the stability of both banks converged to approximately the same range (Figure 4b).

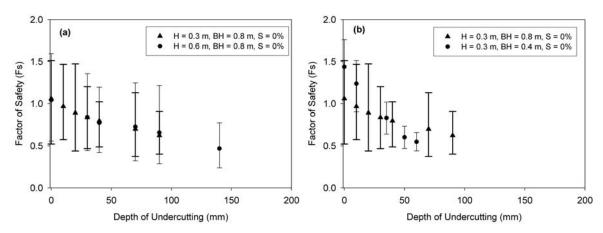


Figure 4. Factor of safety (Fs) versus depth of undercutting for two experiments with same bank height (BH = 0.8 m) and slope (S = 0%) but (a) different constant heads (H = 0.3 m and 0.6 m) and (b) different bank heights (BH = 0.4 m and 0.8 m). Error bars represent minimum and maximum Fs from 2000 Monte Carlo simulations.

The third and fourth categories consist of experiments with the same H and BH but different S, with the differences in S affecting the initial stability of the banks. Banks with a 5% and 10% S were initially unstable (*Fs*<1.0). The difference in the mean *Fs* was approximately the same for all depths of undercutting. However, the difference between the minimum and maximum values tended to decrease as the slope increased (Figure 5).

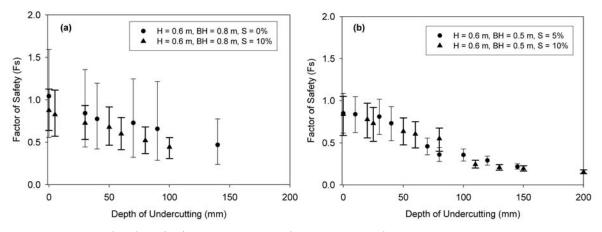


Figure 5. Factor of safety (Fs) versus depth of undercutting for experiments with same constant head (H=0.6 m) and (a) same bank height (BH = 0.8 m) but different slopes (S = 0 and 10%) and (b) same bank height (BH = 0.5 m) but different slopes (S = 5 and 10%). Error bars represent minimum and maximum Fs from 2000 Monte Carlo simulations.

Comparison of the different parameters of the lysimeter experiments showed that the initial stability of the bank was controlled by the BH and the S of the bank. This reflects the basis of the equations used for limit equilibrium; i.e., bank stability when undercutting is not considered is a function of the geometry of the bank and the soil strength parameters. The size of undercutting and the change in the mean *Fs* resulting from seepage was controlled by H. Regardless of the initial stability of the bank, stability quickly converged as undercutting progressed. This convergence made it possible to fit a curve to data from all six lysimeter experiments that suggests an exponential relationship between the depth of undercutting and the mean *Fs* (Figure 6).

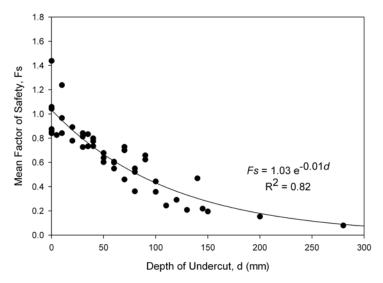


Figure 6. Depth of undercutting, d, versus mean factor of safety, Fs, of all six lysimeter experiments modeled with SLOPE/W.

Conclusion

This research demonstrated a procedure for incorporating seepage undercutting into bank stability models using data from two-dimensional soil lysimeter experiments of seepage erosion. Changes in soil-water pressure were simulated using SEEP/W, a variably saturated numerical flow model, while slope stability was analyzed using SLOPE/W based on limit equilibrium. Undercutting was mimicked by manually changing the geometry of the flow domain for the LS layer based on the measured dimensions, shape and timing of the undercutting due to seepage erosion. The mean *Fs* was used as an index of bank stability for all experiments.

Changes in soil strength, in response to soil-water pressure changes during seepage, were not sufficient to contribute to bank instability, yet the mean *Fs* decreased significantly as the depth of undercutting increased. The decrease in the mean *Fs* due to undercutting was in the range of 42 to 91% depending on the initial stability of the bank. Regardless of the initial stability of the bank, stability converged as undercutting progressed. This means that a stable bank can quickly become unstable when seepage undercutting is considered. For stable banks, the probability of failure reached 100% when the depth of the undercutting reached approximately 30 to 50 mm. Bank height and bank slope controlled the initial stability of the bank while the established constant head controlled the depth of undercutting and the mean *Fs* as undercutting progressed. Based on the results of the lysimeter experiments, the mean *Fs* is exponentially related to the depth of undercutting. These results show that the loss of supporting material

brought about by seepage undercutting can be a major cause of slope instability and may be of equal or greater importance than the impact of increased soil-water pressure on soil strength. This work highlights the need to incorporate the dynamic process of seepage erosion into integrated subsurface flow and streambank stability models

Acknowledgements

This material is based upon work supported by the Cooperative State Research, Education, and Extension Service, U.S. Department of Agriculture, under Award No. 2005-35102-17209. The authors also acknowledge the technical assistance of Onur Akay, Research Engineer, Oklahoma State University, Stillwater, OK.

References

- Abam, T.K.S. 1993. Factors affecting distribution of instability of river banks in the Niger delta. Engineering Geology 35: 123-133.
- Bryan, R.B., and J.A.A. Jones. 1997. The significance of soil piping processes: inventory and prospect. *Geomorphology* 20: 209-218.
- Burgi, P.H., and S. Karaki. 1971. Seepage effect on channel bank stability. *Journal of the Irrigation and Drainage Division* 97: 59-72.
- Caviness, K., G.A. Fox, and P.N. Deliman. 2006. Modeling the Big Black River: Comparison of water quality models. *Journal of the American Water Resources Association (JAWRA)* 42(3): 617-627.
- Crosta, G., and C. di Prisco. 1999. On slope instability induced by seepage erosion. *Canadian Geotechnical Journal* 36: 1056-1073.
- Dapporto, S., M. Rinaldi, N. Casagli, and P. Vannoci. 2003. Mechanisms of riverbank failure along the Arno River, Central Italy. *Earth Surface Processes and Landforms* 28: 1303-1323.
- Darby, S.E., and C.R. Thorne. 1996. Numerical simulation of widening and bed deformation of straight sand-bed rivers. I. Model development. *Journal of Hydraulic Engineering* 122: 184-193.
- Duncan, J.M., and S.G. Wright. 1980. The accuracy of equilibrium methods of slope stability analysis. *Engineering Geology* 16: 5-17.
- Edil, T.B., and L.E. Vallejo. 1980. Mechanics of coastal landslides and the influence of slope parameters. *Engineering Geology* 16: 83-96.
- Fox, G.A., G.V. Wilson, R.K. Periketi, and R.F. Cullum. 2006. Sediment transport model for seepage erosion of streambank sediment. *Journal of Hydrologic Engineering* 11(6): 603-611.
- Fox, G.A., G.V. Wilson, A. Simon, E. Langendoen, O. Akay, and J.W. Fuchs. 2007. Measuring streambank erosion due to ground water seepage: Correlation to bank pore water pressure, precipitation, and stream stage. *Earth Surface Processes and Landforms* DOI: 10.1002/esp.1490.
- Hagerty, D.J., M.F. Spoor, and C.R. Ullrich. 1981. Bank failure and erosion on the Ohio River. *Engineering Geology* 17: 141-158.
- Hagerty, D.J. 1991. Piping/sapping erosion. 1. Basic considerations. *Journal of Hydraulic Engineering* 117(8): 991-1008.
- Huang, C., and J.M. Laflen. 1996. Seepage and soil erosion for a clay loam soil. *Soil Science Society of America Journal* 60(2): 408-416.

- Krahn, J. 2004a. Seepage modeling with SEEP/W: An engineering methodology. GEO-SLOPE International Ltd. Calgary, Alberta, Canada.
- Krahn, J. 2004b. Stability modeling with SLOPE/W: An engineering methodology. GEO-SLOPE/W International Ltd. Calgary, Alberta, Canada.
- Kusabe, O., Y. Okumura, and A. Nakase. 1987. Centrifuge modeling of river bank failures. *Proceedings of International Symposium on Flood Frequency and Risk Analysis, pp.* 399-408.
- Lourenco, S.D.N., K. Sassa, and H. Fukuoka. 2006. Failure process and hydrologic response of a two layer physical model: Implications for rainfall-induced landslides. *Geomorphology* 73: 115-130.
- Morgenstern, N.R., and V.E. Price. 1965. The analysis of the stability of general slip surfaces. *Geotechnique* 15: 79-93.
- Owoputi, L.O., and W.J. Stolte. 2001. The role of seepage in erodibility. *Hydrological Processes* 15(1): 13-22.
- Periketi, R. 2005. Analysis of Seepage Erosion with Lysimeter Experiments and Numerical Modeling. M.S. Thesis, Department of Civil Engineering, University of Mississippi, University, MS.
- Rinaldi M., and N. Casagli. 1999. Stability of streambanks formed in partially saturated soils and effects of negative pore water pressures: The Siene River (Italy). *Geomorphology* 26: 253-277.
- Rockwell, D.L. 2002. The influence of groundwater on surface flow erosion processes. *Earth Surface Processes and Landforms* 27(5): 495-514.
- Simon A., and S.E. Darby. 1997. Disturbance, channel evolution and erosion rates: Hotophia creek, Mississippi. *Proceedings: Conference on Management of Landscapes Disturbed by Channel Incision.* Wang SSY, Langendoen EJ, and Shields FD (eds.), ISBN 0-937099-05-8: 476-481.
- Simon A., A. Curini, S.E. Darby, and E.J. Langendoen. 1999. Streambank mechanics and the role of bank and near-bank processes in incised channels. In *Incised River Channels*, Darby SE, Simon A (Eds), John Wiley and Sons, Chichester, UK.: 193-217.
- van Genuchten, M.T. 1980. A closed-form equation for predicting the hydraulic conductivity of unsaturated soils. *Soil Science Society of America Journal* 44(5): 892-898.
- Wilson G.V., R.K. Periketi, G.A. Fox, S.M. Dabney, F.D. Shields, and R.F. Cullum. 2007. Seepage erosion properties contributing to streambank failure. *Earth Surface Processes and Landforms* 32(3): DOI: 10.1002/esp.1405.